**Virginia’s Revised Mathematics**

**Standards of Learning**

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A Vertical Articulation by Strand

**Algebra I, Geometry, Algebra II**

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**ALGEBRA I**

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| **Expressions and Operations** |
| A.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.* Translate verbal **quantitative situations** into algebraic expressions **and vice versa**.
* **Model real-world situations with algebraic expressions in a variety of representations (concrete, pictorial, symbolic, verbal).**
* **e. Evaluate expressions that contain absolute value, square roots, and cube roots.**
 | A.2 The student will perform operations on polynomials, includinga) applying the laws of exponents to perform operations on expressions;b) adding, subtracting, multiplying, and dividing polynomials; andc) factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.* **Find products of polynomials. The factors will have no more than five total terms. (i.e. (4x+2)(3x+5) represents four terms and (x+1)(2x2+x+3) represents five terms)**
* Find the quotient of polynomials, using a monomial or binomial divisor, **or a completely factored divisor.**
* Factor **completely first- and** second-degree polynomials with integral coefficients
* Identify **prime** polynomials.
 | A.3 The student will express the square roots and cube roots of whole numbers and the square root of a monomial algebraic expression in simplest radical form.* **Express the cube root of a whole number in simplest form.**
* **Express the principal square root of monomial algebraic expression in simplest form.**
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| **Equations and Inequalities** |
| A.4 The student will solve multistep linear and quadratic equations in two variables, includinga) solving literal equations (formulas) for a given variable;b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;c) solving quadratic equations algebraically and graphically;d) solving multistep linear equations algebraically and graphically;e) solving systems of two linear equations in two variables algebraically and graphically; andf) solving real-world problems involving equations and systems of equations.Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.* Confirm algebraic solutions to linear **and quadratic** equations, using a graphing calculator.
* Simplify expressions and solve equations, using the **field** properties **of the real numbers and properties of equality to justify simplification and solution.**
* Write a system of two linear equations that **models** a **real-world** situation.
 | A.5 The student will solve multistep linear inequalities in two variables, includinga) solving multistep linear inequalities algebraically and graphically;b) justifying steps used in solving inequalities, using axioms of inequality and properties oforder that are valid for the set of real numbers and its subsets;c) solving real-world problems involving inequalities; andd) solving systems of inequalities.* Solve multistep **linear** equations.
* **Justify steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers.**
* **Solve systems of linear inequalities algebraically and graphically.**
 | A.6 The student will graph linear equations and linear inequalities in two variables, includinga) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; andb) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.* Graph linear equations and inequalities in two variables, **including those** that arise from a variety of **real-world** situations.
* Use the **parent function** *y* = *x and* **describe** transformations defined by changes in the slope or *y*-intercept.
* **Use transformational graphing to investigate effects of changes in equation parameters on the graph of the equation.**
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| **Functions** |
| A.7 The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including a) determining whether a relation is a function; b) domain and range; c) zeros of a function; d) *x*- and *y*-intercepts; e) finding the values of a function for elements in its domain; and f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.* **Detect patterns in data and represent arithmetic and geometric patterns algebraically.**
* **Detect departures from patterns in data.**
 | **A.8** The student, given a situation in a real-world context, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.* Given a **situation, including a real-world situation,** determine whether a direct variation exists.
* **Given a situation, including a real-world situation, determine whether an inverse variation exists.**
* **Write an equation for an inverse variation, given a set of data.**
* **Graph an equation representing a direct variation, given a set of data.**
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| **Statistics** |
| **A.9 The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.*** **Given data, including data in a real-world context, calculate and interpret the mean absolute deviation.**
* **Given data, including data in a real-world context, calculate and interpret the variance and standard deviation.**
* **Given data, including data in a real-world context, calculate and interpret z-scores for the data.**
* **Analyze descriptive statistics to determine the implications for the real-world situations from which the data derive.**
* **Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.**
* **Compare and contrast mean absolute deviation and standard deviation in a real-world context.**
 | A.10 The student will compare and contrast multiple univariate data sets using box-and-whisker plots. * Compare, **contrast**, and analyze data, **including data from real-world situations displayed in** box-and-whisker plots.
 | A11 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world problems using mathematical models. Mathematical models will include linear and quadratic functions.* Write an equation for **a curve** of best fit, given a set of **no more than twenty** data points in a table, a graph, or a **real-world** situation.
* Make predictions about unknown outcomes, using the equation of **the curve** of best fit.
* **Design experiments and collect data to address specific, real-world questions.**
* **Evaluate the contextual validity of a mathematical model of a real-world situation.**
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**GEOMETRY**

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| **REASONING, LINES, AND TRANSFORMATIONS** |
| G.1 The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include a) identifying the converse, inverse, and contrapositive of a conditional statement; b) translating a short verbal argument into symbolic form; c) using Venn diagrams to represent set relationships; and d) using deductive reasoning.* Use valid forms of deductive reasoning, including the law of syllogism, **the law of the contrapositive, the law of detachment, and counterexamples.**
* **Use Venn diagrams to represent set relationships, such as intersection, and union.**
* **Interpret Venn diagrams.**
* **Recognize and use the symbols of formal logic**
 | G.2 The student will use the relationships between angles formed by two lines cut by a transversal to a) determine whether two lines are parallel; b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.* Use algebraic~~,~~ **and** coordinate **methods as well as** deductive **proofs** to **verify** whether two lines are parallel.
* Solve **real-world** problems involving intersecting and parallel lines in a plane.
 | G.3 The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include a) investigating and using formulas for finding distance, midpoint, and slope; b) applying slope to verify and determine whether lines are parallel or perpendicular; c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.* **Compare the slopes to determine whether two lines are parallel, perpendicular, or neither.**
* Given an image and preimage, identify the transformation that has taken place as a reflection, rotation, **dilation,** or translation.
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| **REASONING, LINES, AND TRANSFORMATIONS** |
| **G.4** The student will construct and justify the constructions of a) a line segment congruent to a given line segment; b) the perpendicular bisector of a line segment; c) a perpendicular to a given line from a point not on the line; d) a perpendicular to a given line at a given point on the line; e) the bisector of a given angle; f) an angle congruent to a given angle; g) a line parallel to a given line through a point not on the given line. * Construct **and justify the constructions of**
	+ a line segment congruent to a given line segment;
	+ the **perpendicular** bisector of a line segment;
	+ a perpendicular to a given line from a point not on the line;
	+ a perpendicular to a given line at a point on the line;
	+ the bisector of a given angle;
	+ an angle congruent to a given angle; **and**

**– a line parallel to a given line through a point not on the given line.** |  |  |
| **TRIANGLES** |
| **G.5** The student, given information concerning the lengths of sides and/or measures of angles in triangles, will a) order the sides by length, given the angle measures; b) order the angles by degree measure, given the side lengths; c) determine whether a triangle exists; and d) determine the range in which the length of the third side must lie. These concepts will be considered in the context of real-world situations.* **Order the sides of a triangle by their lengths when given the measures of the angles.**
* **Solve real-world problems given information about the lengths of sides and/or measures of angles in triangles.**
 | **G.6** The student, given information in the form of a figure or statement, will prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.* Use definitions, postulates, and theorems to **prove** triangles congruent.
* **Use algebraic methods to prove two triangles are congruent.**

**G.7** The student, given information in the form of a figure or statement, will prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.* Use definitions, postulates, and theorems to **prove** triangles similar.
 | **G.8** The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.* Solve **real-world** problems, using right triangle trigonometry and properties of right triangles.
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| **POLYGONS AND CIRCLES** |
| **G.9** The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.* Solve problems, **including real-world problems**, using the properties specific to parallelograms, rectangles, rhombi, squares, **isosceles trapezoids**, and trapezoids.
* Prove the **characteristics** of quadrilaterals, using deductive reasoning, algebraic, and coordinate methods.
 | **G.10** The student will solve real-world problems involving angles of polygons.* Solve **real-world** problems involving the measures of interior and exterior angles of polygons.
* **Find the number of sides of a regular polygon, given the measures of interior or exterior angles of the polygon.**
 | **G.11** The student will use angles, arcs, chords, tangents, and secants to a) investigate, verify, and apply properties of circles; b) solve real-world problems involving properties of circles; and c) find arc lengths and areas of sectors in circles.* Find lengths, **angle measures, and arc measures associated with**

– two intersecting chords; – **two intersecting secants;** **– an intersecting secant and tangent;** – two intersecting tangents; **and** **– central and inscribed angles.*** Calculate the area of a sector **and the length of an arc** of a circle, using proportions.
* Solve **real-world** problems associated with circles, using properties of angles, **lines**, and arcs.
* **Prove properties of circles, using deductive reasoning, algebraic, and coordinate methods.**
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| **POLYGONS AND CIRCLES** |
| **G.12 The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.*** **Identify the center of a circle from a given standard equation.**
* **Use the distance formula to find the radius of a circle.**
* **Given the coordinates of the center and radius of the circle, identify a point on the circle.**
* **Given the equation of a circle, identify the coordinates of the center and find the radius of the circle.**
* **Given the coordinates of the endpoints of a diameter, find the equation of the circle.**
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| **THREE-DIMENSIONAL FIGURES** |
| **G.13** The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.* Solve problems, **including real-world problems**, involving total surface area and volume of cylinders, prisms, pyramids, cones, and spheres as well as combinations of three-dimensional figures.
* **Calculators may be used to find decimal approximations for results.**
 | **G.14** The student will use similar geometric objects in two- or three-dimensions to a) compare ratios between side lengths, perimeters, areas, and volumes; b) determine how changes in one or more dimensions of an object affect area and/or volume of the object; c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and d) solve real-world problems about similar geometric objects.* Describe how a change in one **or more** dimensions affect~~s~~ other **derived** measures (perimeter, area, total surface area, and volume) of an object.
* **Describe how changes in one or more measures (perimeter, area, total surface area, and volume) affect other measures of an object.**
* Solve **real-world** problems involving **measured attributes of** similar objects.
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**ALGEBRA II**

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| **EXPRESSIONS and OPERATIONS** |
| **AII.1** The student will a) add, subtract, multiply, divide, and simplify rational algebraic expressions; b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; c) write radical expressions as expressions containing rational exponents and vice versa; and d) factor polynomials completely.* Add, subtract, multiply, and divide rational **algebraic** expressions
* Simplify a rational **algebraic** expression with common monomial or binomial factors.
* Recognize a complex **algebraic** fraction, and simplify it as a quotient or product of simple **algebraic** fractions.
* **Factor polynomials by applying general patterns including** difference of squares, sum and difference of cubes, and perfect square trinomials.
* **Factor polynomials completely over the integers**
 | **AII.3** The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of *i*, and identify field properties that are valid for the complex numbers.* **Determine which field properties apply to the complex number system.**
* **Simplify radical expressions containing negative rational numbers and express in a+b*i* form.**
* **Write a real number in a+b*i* form.**
* **Write a pure imaginary number in a+b*i* form.**
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| **RELATIONS and FUNCTIONS** |
| **AII.2 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first *n* terms, finding the *n*th term, and evaluating summation formulas. Notation will include** Σ **and *an*.*** **Generalize patterns in a sequence using explicit and recursive formulas.**
* **Use finite geometric series to model real-world situations.**
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| **EQUATIONS and INEQUALITIES** |
| **AII.4** The student will solve, algebraically and graphically, a) absolute value equations and inequalities; b) quadratic equations over the set of complex numbers; c) equations containing rational algebraic expressions; and d) equations containing radical expressions. Graphing calculators will be used for solving and for confirming the algebraic solutions.* **Solve absolute value equations and inequalities algebraically and graphically.**
* Solve a quadratic equation over the set of complex numbers **using an appropriate strategy.**
* Solve equations containing rational algebraic expressions with monomial **or binomial** denominators algebraically and graphically.
* **Verify** possible solutions to an equation containing rational or radical expressions.
* **Apply an appropriate equation to solve a real-world problem.**
 | **AII.5** The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.* **Predict the number of solutions to a nonlinear system of two equations.**
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| **FUNCTIONS** |
| **AII.6** The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.* **Recognize graphs of parent functions.**
* **Given the equation and using a transformational approach, graph a function.**
* **Given the graph of a function, identify the parent function.**
* **Given the graph of a function, identify the transformations that map the image to the preimage in order to determine the equation of the image.**
* **Using a transformational approach, write the equation of a function given its graph.**
 | **AII.7** The student will investigate and analyze functions algebraically and graphically. Key concepts include a) domain and range, including limited and discontinuous domains and ranges; b) zeros; c) *x*- and *y*-intercepts; d) intervals in which a function is increasing or decreasing; e) asymptotes; f) end behavior; g) inverse of a function; and h) composition of multiple functions. Graphing calculators will be used as a tool to assist in investigation of functions.* **Given the graph of a function, identify intervals on which the function is increasing and decreasing.**
* **Describe the end behavior of a function.**
* **Describe** restricted/ discontinuous domains and ranges.
* **Find the inverse of a function.**
* **Graph the inverse of a function as a reflection across the line *y* = *x*.**
* **Use composition of functions to verify two functions are inverses.**
* **Convert between logarithmic and exponential forms of an equation.**
* **Find the equations of vertical and horizontal asymptotes of functions**
 | **AII.8** The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.* **Define a polynomial function, given its zeros.**
* **Determine the factored form of a polynomial expression from the x-intercepts of the graph of its corresponding function.**
* **For a function, identify zeros of multiplicity greater than 1 and describe the effect of those zeros on the graph of the function.**
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| **STATISTICS** |
| **AII.9** The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.* Find an equation for the curve of best fit for data, using a graphing calculator. Models will include **polynomial**, exponential, and logarithmic functions.
* Make predictions, using data, scatterplots, or **the equation of the** curve of best fit.
 | **AII.10** The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.* **Given a situation,** determine the value of the constant of proportionality
* Set up and solve **problems, including real-world problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.**
 | **AII.11** The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.* **Identify the properties of a normal probability distribution.**
* **Describe how the standard deviation and the mean affect the graph of the normal distribution.**
* **Compare two sets of normally distributed data using a standard normal curve and z-scores.**
* **Represent probability as area under the curve of a standard normal probability distribution.**
* **Use the graphing calculator or a standard normal probability table to determine probabilities or percentiles based on z-scores.**
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| **STATISTICS** |
| **AII.12** The student will compute and distinguish between permutations and combinations and use technology for applications.* **Compare and contrast permutations and combinations.**
* **Calculate the number of permutations of *n* objects taken *r* at a time.**
* **Calculate the number of combinations of *n* objects taken *r* at a time.**
* **• Use permutations and combinations as counting techniques to solve real-world problems.**
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Analysis of the 2009 Math Standards by Strand

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**Content that Remains Unchanged:**

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**Content that has been Introduced/Adjusted:**

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