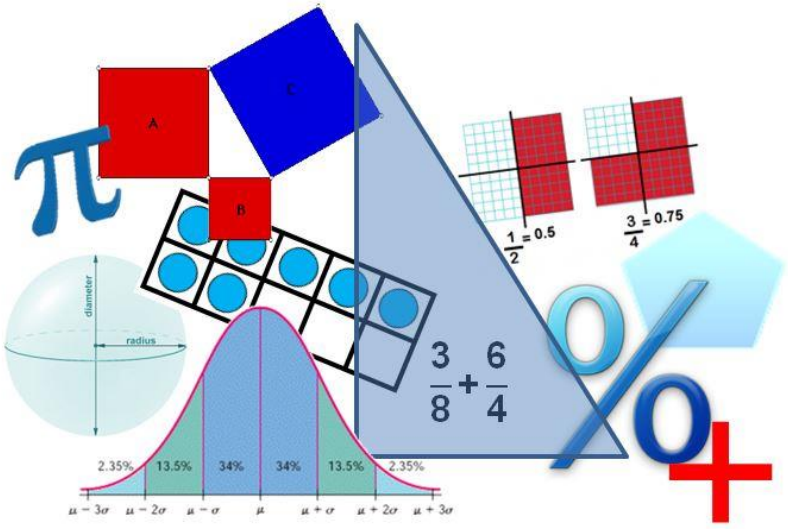
Mathematics

2016 Standards of Learning

Mathematical Analysis Curriculum Framework



Board of Education
Commonwealth of Virginia

Superintendent of Public Instruction
Steven R. Staples

Chief Academic Officer/Assistant Superintendent for Instruction
Steven M. Constantino

Office of Mathematics and Governor's Schools
Debra Delozier, Mathematics Specialist
Tina Mazzacane, Mathematics and Science Specialist
Christa Southall, Mathematics Specialist

Acknowledgements
The Virginia Department of Education wishes to express sincere thanks to Michael Bolling, who assisted in the development of the 2016 *Mathematics Standards of Learning* and 2016 *Mathematics Standards of Learning Curriculum Framework*.

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills

This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "… the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

MA.1 The student will investigate and identify the properties of polynomial, rational, piecewise, and step functions and sketch the graphs of the functions.

Understanding the Standard	Essential Knowledge and Skills
• The graph of polynomial, rational, piecewise, and step functions can be determined by exploring properties of the functions. • A variety of notations should be used, including set notation and interval notation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a polynomial, rational, piecewise, and step function, given an equation or graph. • Given a graph or equation of a polynomial, rational, piecewise, or step function, identify: – domain and range; – zeros; – intercepts; – symmetry; – asymptotes (horizontal, vertical, and oblique/slant); – points of discontinuity; – intervals for which the function is increasing, decreasing or constant; – end behavior; and – relative and/or absolute maximum and minimum points. • Sketch the graph of a polynomial, rational, piecewise, or step function. • Investigate and verify characteristics of a polynomial, rational, piecewise, and step function, using a graphing utility. • Rationalize the denominator of a rational function.

MA.2 The student will investigate and identify the characteristics of exponential and logarithmic functions to graph the function, solve equations, and solve practical problems.

Understanding the Standard	Essential Knowledge and Skills
• Exponential and logarithmic functions are inverse functions. • Some examples of appropriate models or situations for exponential and logarithmic functions are: – population growth; – compound interest; – depreciation/appreciation; – Richter scale; and – radioactive decay.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify exponential functions from an equation or a graph. • Identify logarithmic functions from an equation or a graph. • Define *e*, and know its approximate value. • Convert between equations written in logarithmic and exponential form. • Identify common and natural logarithms, given an equation or practical situation. • Use laws of exponents and logarithms to solve equations and simplify expressions. • Model practical problems, using exponential and logarithmic functions. • Graph exponential and logarithmic functions and identify asymptotes, end behavior, intercepts, domain, and range.

MA.3 The student will apply compositions of functions and inverses of functions to practical situations and investigate and verify the domain and range of resulting functions.

Understanding the Standard	Essential Knowledge and Skills
• In composition of functions, a function serves as input for another function. • The composition of functions $f(x)$ and $g(x)$ can be determined using the graphs of $f(x)$ and $g(x)$. • A graph of a function and its inverse are symmetric about the line $y = x$. • $\left(f \circ f^{-1}\right)(x) = \left(f^{-1} \circ f\right)(x) = x$	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the composition of functions algebraically and graphically. • Determine the inverse of a function algebraically and graphically. • Determine the domain and range of composite functions algebraically and graphically. • Determine the domain and range of the inverse of a function algebraically and graphically.

MA.4 The student will determine the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity.

Understanding the Standard	Essential Knowledge and Skills
• The limit of a function is the value approached by $f(x)$ as x approaches a given value or infinity.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Verify estimates about the limit of a function using a graphing utility. • Determine the limit of a function algebraically and verify with a graphing utility. • Determine the limit of a function numerically and verify with a graphing utility. • Use limit notation when describing end behavior of a function.

MA.5 The student will investigate and describe the continuity of functions.

Understanding the Standard	Essential Knowledge and Skills
• Continuous and discontinuous functions can be identified by their equations or graphs. • Discontinuity can be described as point (removable), jump or infinite. • A function, f, is continuous at a point c if and only if – $f(c)$ exists – $\lim_{x\to c} f(x)$ exists – $f(c) = \lim_{x\to c} f(x)$	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe continuity of a function. • Investigate the continuity of functions including absolute value, step, rational, and piecewise functions, using graphical and algebraic methods. • Classify types of discontinuity. • Prove continuity at a point, using the definition of limits.

MA.6 The student will investigate, graph, and identify the properties of conic sections from equations in vertex and standard form.

Understanding the Standard	Essential Knowledge and Skills
• Matrices can be used to represent transformations of figures in the plane. • A conic section is a figure formed by the intersection of a plane with a right circular cone. Conic sections include the parabola, circle, ellipse, and hyperbola.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a translation or rotation matrix, determine an equation for the transformed function or conic section. • Investigate and verify graphs of transformed conic sections, using a graphing utility. • Graph conic sections from equations written in vertex or standard form using transformations. • Identify properties of conic sections.

MA.7 The student will perform operations with vectors in the coordinate plane and solve practical problems using vectors.

Understanding the Standard	Essential Knowledge and Skills
• Every vector has an equal vector that has its initial point at the origin. • The magnitude and direction of a vector with the origin as the initial point are completely determined by the coordinates of its terminal point. • Every nonzero vector has a corresponding unit vector, which has the same direction as the vector but a magnitude of 1.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use vector notation. • Perform the operations of addition, subtraction, scalar multiplication, and inner (dot) product on vectors. • Graph vectors and resultant vectors. • Express complex numbers in vector notation. • Identify properties of vector addition, scalar multiplication, and dot product. • Determine the components of a vector. • Determine the norm (magnitude) of a vector. • Use vectors in simple geometric proofs. • Solve problems, including practical problems, using vectors.

MA.8 The student will identify, create, and solve practical problems involving triangles.

Understanding the Standard	Essential Knowledge and Skills
• Practical problems can be modeled using trigonometry and vectors. • Triangles can be formed from vectors.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve and create problems, including practical problems, using trigonometric functions. • Solve and create problems, including practical problems, using the Pythagorean Theorem. • Solve and create problems, including practical problems, using the Law of Sines and the Law of Cosines. • Solve problems, including practical problems, where triangles are formed from vectors.

MA.9 The student will investigate and identify the characteristics of the graphs of polar equations.

Understanding the Standard	Essential Knowledge and Skills
• The real number system is represented geometrically on the number line, and the complex number system is represented geometrically on the plane where $a + bi$ corresponds to the point (a, b) in the plane.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Classify polar equations (rose, cardioid, limaçon, lemniscate, spiral, and circle), given the graph or the equation. • Determine the effects of changes in the parameters of polar equations on the graph, using a graphing utility. • Convert between complex numbers written in rectangular form and polar form. • Determine and verify the intersection of the graphs of two polar equations, using a graphing utility.

MA.10 The student will use parametric equations to model and solve practical problems.

Understanding the Standard	Essential Knowledge and Skills
• Parametric equations are used to express two dependent variables, *x* and *y*, in terms of an independent variable (parameter), *t*. • Some curves cannot be represented as a function, *f*(*x*). Parametric graphing enables the representation of these curves in terms of functions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Graph parametric equations. • Use parametric equations to model practical problems, including motion over time. • Determine solutions to parametric equations graphically. • Use a graphing utility to graph and analyze parametric equations.

MA.11 The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.

Understanding the Standard	Essential Knowledge and Skills
• Matrices may be used to solve systems of equations. • Matrices can model a variety of linear systems. • Solutions of a linear system are values that satisfy every equation in the system. • Matrices can be used to model and solve real-world problems.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Multiply matrices by a scalar. • Add, subtract, and multiply matrices. • Model problems with a system of no more than three linear equations. • Express a system of linear equations as a matrix equation. • Solve a system of equations using matrices. • Determine the inverse of a two-by-two or three-by-three matrix using paper and pencil. • Verify two matrices are inverses using matrix multiplication. • Verify the commutative and associative properties for matrix addition and multiplication.

MA.12 The student will expand binomials having positive integral exponents.

Understanding the Standard	Essential Knowledge and Skills
• The Binomial Theorem provides a formula for calculating the product $(a + b)^n$ for any positive integer n. • Pascal's Triangle is a triangular array of binomial coefficients.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Expand binomials having positive integral exponents. • Use the Binomial Theorem, the formula for combinations, and Pascal's Triangle to expand binomials.

MA.13 The student will determine the sum of finite and infinite convergent series.

Understanding the Standard	Essential Knowledge and Skills
• Examination of infinite sequences and series may lead to a limiting process. • Arithmetic sequences have a common difference between any two consecutive terms. • Geometric sequences have a common ratio between any two consecutive terms.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use and interpret the notation: $\sum$, n, nth, and a_n. • Derive the formulas associated with arithmetic and geometric sequences and series. • Given the formula, determine the nth term, a_n, for an arithmetic or geometric sequence. • Given the formula, determine the sum, S_n, if it exists, of an arithmetic or geometric series. • Model and solve problems, using sequence and series information. • Distinguish between a convergent and divergent series. • Discuss convergent series in relation to the concept of a limit.

MA.14 The student will use mathematical induction to prove formulas and mathematical statements.

Understanding the Standard	Essential Knowledge and Skills
• Mathematical induction is a method of proof that depends on a recursive process. • Mathematical induction allows reasoning from specific true values of the variable to general values of the variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare inductive and deductive reasoning. • Prove formulas and mathematical statements, using mathematical induction.