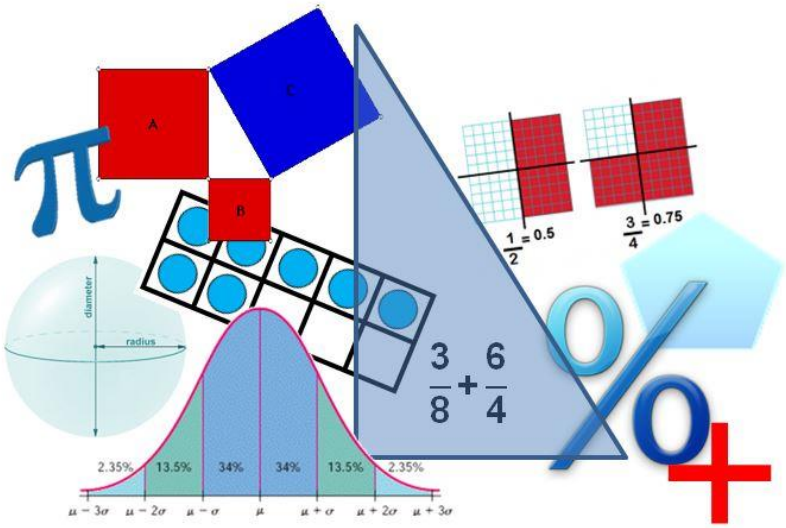
Mathematics

2016 Standards of Learning

Trigonometry
Curriculum Framework



Board of Education
Commonwealth of Virginia

Superintendent of Public Instruction
Steven R. Staples

Chief Academic Officer/Assistant Superintendent for Instruction
Steven M. Constantino

Office of Mathematics and Governor's Schools
Debra Delozier, Mathematics Specialist
Tina Mazzacane, Mathematics and Science Specialist
Christa Southall, Mathematics Specialist

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Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills

This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "… the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

T.1 The student, given a point on the terminal side of an angle in standard position, or the value of the trigonometric function of the angle, will determine the sine, cosine, tangent, cotangent, secant, and cosecant of the angle.

Understanding the Standard	Essential Knowledge and Skills
• Triangular trigonometric function definitions are related to circular trigonometric function definitions. • Both degrees and radians are units for measuring angles. • Drawing an angle in standard position will force the terminal side to lie in a specific quadrant or axis. • A point on the terminal side of an angle determines a reference triangle from which the values of the six trigonometric functions may be derived. • If one trigonometric function value is known, then a triangle can be formed to use in determining the other five trigonometric function values.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define the six triangular trigonometric functions of an angle in a right triangle. • Draw a reference right triangle when given a point on the terminal side of the angle in standard position. • Draw a reference right triangle when given the value of a trigonometric function of the angle. • Determine the value of any trigonometric function when given a point on the terminal side of an angle in standard position. • Given one trigonometric function value, determine the other five trigonometric function values.

T.2 The student will develop and apply the properties of the unit circle in degrees and radians.

Understanding the Standard	Essential Knowledge and Skills
• Triangular trigonometric function definitions are related to circular trigonometric function definitions. • Knowledge of the unit circle is a useful tool for determining all six trigonometric values for special angles (30°, 45°, 60°, and 90°). • The relationships between the angle measures and side lengths of special right triangles (30°-60°-90° and 45°-45°-90°) are widely used in mathematics. • Special right triangles may be used to develop the unit circle. • Unit circle properties will allow special angle and related angle trigonometric values to be found without the aid of a graphing utility. • Degrees and radians are units of angle measure. • A radian is the measure of the central angle that is determined by an arc whose length is the same as the radius of the circle. • There is a connection between sides and angles of special right triangles, the unit circle, and the coordinate plane.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define the six circular trigonometric functions of an angle in standard position. • Apply the properties of the unit circle to determine trigonometric function values of special angles and their related angles in both degrees and radians without using a graphing utility. • Apply the properties of the unit circle to convert between special angles expressed in radians and degrees, without using a graphing utility.

T.3 The student, given one of the six trigonometric functions in standard form, will

a) state the domain and the range of the function;

b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;

c) sketch the graph of the function by using transformations for at least a two-period interval; and

d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

Understanding the Standard	Essential Knowledge and Skills
• The domain and range of a trigonometric function determine the scales of the axes for the graph of the trigonometric function. • The amplitude, period, phase shift, and vertical shift are important characteristics of the graph of a trigonometric function, and each has a specific purpose in applications using trigonometric equations. • The graph of a trigonometric function can be used to display information about the periodic behavior of a real-world situation, such as wave motion or the motion of a Ferris wheel. • Standard form of the trigonometric functions may be written in multiple ways (e.g., $y = A \sin (Bx + C) + D$ or $y = A \sin [B(x + C)] + D$).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • State the domain and the range of a trigonometric function written in standard form. • Determine the amplitude, period, phase shift, vertical shift, and asymptotes of a trigonometric function from the equation of the function and from the graph of the function. • Describe the effect of changing A, B, C, or D in the standard form of a trigonometric equation. • Sketch the graph of a function written in standard form by using transformations for at least a two-period interval, including both positive and negative values for the domain.

T.4 The student will graph the six inverse trigonometric functions.

Understanding the Standard	Essential Knowledge and Skills
• Trigonometric functions are not invertible, because they are periodic. Domain restrictions on trigonometric functions are necessary in order to determine the inverse trigonometric function.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the domain and range of the inverse trigonometric functions. • Use the restrictions on the domains of the inverse trigonometric functions in determining the values of the inverse trigonometric functions. • Graph inverse trigonometric functions.

T.5 The student will verify basic trigonometric identities and make substitutions, using the basic identities.

Understanding the Standard	Essential Knowledge and Skills
• Trigonometric identities can be used to simplify trigonometric expressions, equations, or identities. • Trigonometric identity substitutions can help solve trigonometric equations, verify another identity, or simplify trigonometric expressions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities. The basic trigonometric identities include – reciprocal identities; – Pythagorean identities; – sum and difference identities; – double-angle identities; and – half-angle identities.

T.6 The student will solve trigonometric equations and inequalities.

Understanding the Standard	Essential Knowledge and Skills
• Solutions for trigonometric equations will depend on the domains. • A graphing utility can be used to determine the solution of a trigonometric equation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve trigonometric equations with and without restricted domains algebraically and graphically. • Solve trigonometric inequalities algebraically and graphically. • Verify algebraic solutions, using a graphing utility.

T.7 The student will determine the value of any trigonometric function and inverse trigonometric function.

Understanding the Standard	Essential Knowledge and Skills
• The trigonometric function values of any angle can be found by using a graphing utility. • The inverse trigonometric functions can be used to determine angle measures whose trigonometric function values are known. • Calculations of inverse trigonometric function values can be related to the triangular definitions of the trigonometric functions. • Trigonometric functions are not invertible, because they are periodic. Domain restrictions on trigonometric functions are necessary in order to determine the inverse trigonometric function.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use a graphing utility to determine the trigonometric function values of any angle in either degrees or radians. • Define inverse trigonometric functions. • Determine angle measures by using the inverse trigonometric functions when the trigonometric function values are given.

T.8 The student will create and solve practical problems involving triangles.

Understanding the Standard	Essential Knowledge and Skills
• A practical problem may be solved by using one of a variety of techniques associated with triangles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Create and solve practical problems involving triangles. • Use the trigonometric functions, Pythagorean Theorem, Law of Sines, and Law of Cosines to solve practical problems. • Use the trigonometric functions to model practical situations. • Identify a solution technique associated with triangles that could be used with a given problem. • Apply the sum and difference identities for sine, cosine, and tangent to solve problems.

T.9 The student will solve problems, including practical problems, involving

a) arc length and area of sectors in circles using radians and degrees; and

b) linear and angular velocity.

Understanding the Standard	Essential Knowledge and Skills
• Degrees and radians are units of angle measure. • A radian is the measure of the central angle that is determined by an arc whose length is the same as the radius of the circle. • The relationship between the radian measure of an angle and the length of the intercepted arc can be represented by $s = r\theta$, where s is the arc length, r is the length of the radius, and θ is the measure of the angle.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Convert between any angle expressed in radians and degrees without using a graphing utility. (a) • Derive the relationship between the radian measure of an angle and the length of the intercepted arc. (a) • Calculate the length of an arc in radians. (a) • Calculate the area of sectors in circles. (a) • Solve practical problems involving linear and angular velocity. (b)